Using Law of Sines:

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$c = \frac{a}{\sin\alpha} \times \sin\gamma$$

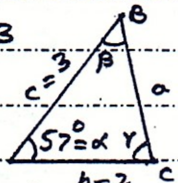
$$c = \frac{32}{\sin 30^\circ} \times \sin 59^\circ 30'$$

$$\boxed{c = 55.14}$$

Q.11 Let the Sides are 2 and 3

$a = ?$ $b = 2$ $c = 3$

$\alpha = 57^\circ$ $\beta = ?$ $\gamma = ?$



Using $\alpha + \beta + \gamma = 180^\circ$

$$\beta + \gamma = 180^\circ - 57^\circ$$

$$\beta + \gamma = 123^\circ \quad \text{(1)}$$

Now $\dfrac{b-c}{b+c} = \dfrac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)}$

$$\frac{2-3}{2+3} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{123^\circ}{2}\right)}$$

$$\tan\left(\frac{\beta-\gamma}{2}\right) = \frac{-1}{5} \times \tan\left(\frac{123^\circ}{2}\right)$$

$$\tan\left(\frac{\beta-\gamma}{2}\right) = -0.3684$$

$$\frac{\beta-\gamma}{2} = -20^\circ 13'$$

$$\beta - \gamma = -40^\circ 26' \quad \text{(2)}$$

Adding (1) and (2)

$$2\beta = 82^\circ 34'$$

$$\boxed{\beta = 41^\circ 17'}$$

$(1) \Rightarrow \gamma = 123^\circ - \beta$

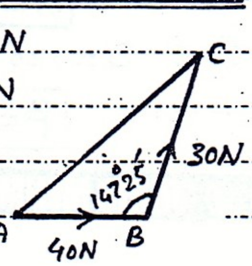
$$\gamma = 123^\circ - 41^\circ 17'$$

$$\boxed{\gamma = 81^\circ 43'}$$

Q.12 $|\vec{AB}| = 40N$

$|\vec{BC}| = 30N$

Angle b/w forces $= 147^\circ 25'$



Using Law of Cosines:

$$(\vec{AC})^2 = (\vec{AB})^2 + (\vec{BC})^2 - 2(\vec{AB})(\vec{BC})\cos 147^\circ 25'$$

$$(\vec{AC})^2 = (40)^2 + (30)^2 - 2(40)(30)\cos(147^\circ 25')$$

$$(\vec{AC})^2 = 4522.26$$

$$|\vec{AC}| = 67.25\,N$$

EXERCISE: 12.6

Q.1 $a = 7$ $b = 7$ $c = 9$

$\alpha = ?$ $\beta = ?$ $\gamma = ?$

Using $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

$$= \frac{7^2 + 9^2 - 7^2}{2(7)(9)} = 0.6429$$

$$\alpha = \cos^{-1}(0.6429)$$

$$\boxed{\alpha = 50^\circ}$$

Using $\cos\beta = \dfrac{c^2 + a^2 - b^2}{2ca}$

$$= \frac{9^2 + 7^2 - 7^2}{2(9)(7)} = 0.6429$$

$$\beta = \cos^{-1}(0.6429)$$

$$\boxed{\beta = 50^\circ}$$

Now $\alpha + \beta + \gamma = 180^\circ$

$$\gamma = 180^\circ - 50^\circ - 50^\circ$$

$$\boxed{\gamma = 80^\circ}$$

Q.2 $a = 32$ $b = 40$ $c = 66$

$\alpha = ?$ $\beta = ?$ $\gamma = ?$

Using $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

$$= \frac{(40)^2 + (66)^2 - (32)^2}{2(40)(66)}$$

$$\cos\alpha = 0.9341$$

$$\alpha = \cos^{-1}(0.9341)$$

$$\boxed{\alpha = 20^\circ 55'}$$

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Using
$$\cos\beta = \frac{c^2+a^2-b^2}{2ca}$$
$$= \frac{(66)^2+(32)^2-(40)^2}{2(66)(32)}$$
$$\cos\beta = 0.8949$$
$$\boxed{\beta = 26^\circ 30'}$$
$$\because \alpha+\beta+\gamma = 180^\circ$$
$$\gamma = 180^\circ - 20^\circ 55' - 26^\circ 30'$$
$$\boxed{\gamma = 132^\circ 35'}$$

Q.3 $a = 28.3 \quad b = 31.7 \quad c = 42.8$

$\alpha = ? \quad \beta = ? \quad \gamma = ?$

Using $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$
$$\cos\alpha = \frac{(31.7)^2+(42.8)^2-(28.3)^2}{2(31.7)(42.8)}$$
$$\cos\alpha = 0.7503$$
$$\boxed{\alpha = 41^\circ 23'}$$
Using $\cos\beta = \dfrac{c^2+a^2-b^2}{2ca}$
$$\cos\beta = \frac{(42.8)^2+(28.3)^2-(31.7)^2}{2(42.8)(28.3)}$$
$$\cos\beta = 0.6720$$
$$\boxed{\beta = 47^\circ 47'}$$
$$\because \alpha+\beta+\gamma = 180^\circ$$
$$\gamma = 180^\circ - \alpha - \beta$$
$$\gamma = 180^\circ - 41^\circ 23' - 47^\circ 47'$$
$$\boxed{\gamma = 90^\circ 50'}$$

Q.4 $a = 31.9 \quad b = 56.31 \quad c = 40.27$

$\alpha = ? \quad \beta = ? \quad \gamma = ?$

Using $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$
$$\cos\alpha = \frac{(56.31)^2+(40.27)^2-(31.9)^2}{2(56.31)(40.27)}$$
$$\cos\alpha = 0.8323$$
$$\boxed{\alpha = 33^\circ 40'}$$
Using $\cos\beta = \dfrac{c^2+a^2-b^2}{2ca}$
$$\cos\beta = \frac{(40.27)^2+(31.9)^2-(56.31)^2}{2(40.27)(31.9)^2}$$
$$\cos\beta = -0.2069$$
$$\boxed{\beta = 101^\circ 56'}$$
$$\because \alpha+\beta+\gamma = 180^\circ$$
$$\gamma = 180^\circ - \alpha - \beta$$
$$\gamma = 180^\circ - 33^\circ 40' - 101^\circ 56'$$
$$\boxed{\gamma = 44^\circ 24'}$$

Q.5 $a = 4584 \quad b = 5140 \quad c = 3624$

$\alpha = ? \quad \beta = ? \quad \gamma = ?$

Using $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$
$$\cos\alpha = \frac{(5140)^2+(3624)^2-(4584)^2}{2(5140)(3624)}$$
$$\cos\alpha = 0.4977$$
$$\boxed{\alpha = 60^\circ 9'}$$
Using $\cos\beta = \dfrac{c^2+a^2-b^2}{2ca}$
$$\cos\beta = \frac{(3624)^2+(4584)^2-(5140)^2}{2(3624)(4584)} = 0.2326$$
$$\beta = \cos^{-1}(0.2326)$$
$$\boxed{\beta = 76^\circ 33'}$$

$\because \alpha+\beta+\gamma=180^\circ$

$\gamma = 180^\circ - \alpha - \beta$

$\gamma = 180^\circ - 76^\circ 33' - 60^\circ 9'$

$\boxed{\gamma = 43^\circ 18'}$

Q.6 *Find Smallest angle:*

$a = 37.34 \quad b = 3.24 \quad c = 35.06$

$\because$ b is smallest side so β will be smallest angle.

Using $\cos\beta = \dfrac{c^2+a^2-b^2}{2ca}$

$\cos\beta = \dfrac{(35.06)^2+(37.34)^2-(3.24)^2}{2(35.06)(37.34)}$

$\cos\beta = 0.9980$

$\beta = \cos^{-1}(0.9980)$

$\boxed{\beta = 3^\circ 38' 45''}$

is smallest angle.

Q.7 *Find greatest angle:*

$a = 16 \quad b = 20 \quad c = 33$

$\because$ c is largest side so γ will be greatest angle.

Using $\cos\gamma = \dfrac{a^2+b^2-c^2}{2ab}$

$\cos\gamma = \dfrac{(16)^2+(20)^2-(33)^2}{2(16)(20)}$

$\cos\gamma = -0.6766$

$\gamma = \cos^{-1}(-0.6766)$

$\boxed{\gamma = 132^\circ 34'}$

Imp

Q.8 $a = x^2+x+1 \quad b = 2x+1 \quad c = x^2-1$

$\because$ a is greatest side so α will be greatest angle.

using $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$\cos\alpha = \dfrac{(2x+1)^2+(x^2-1)^2-(x^2+x+1)^2}{2[x^2-1][2x+1]}$

$= \dfrac{\cancel{4x^2}+4x+\cancel{1}+\cancel{x^4}+1-\cancel{2x^2}-\cancel{x^4}-x^2-\cancel{1}-2x^3-2x-\cancel{2x^2}}{2[2x^3+x^2-2x-1]}$

$= \dfrac{-2x^3-x^2+2x+1}{2[2x^3+x^2-2x-1]}$

$\cos\alpha = \dfrac{-1\cancel{[2x^3+x^2-2x-1]}}{2\cancel{[2x^3+x^2-2x-1]}}$

$\cos\alpha = -1/2$

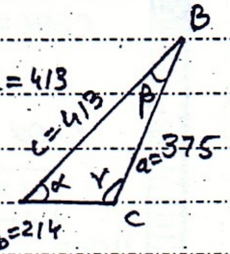
$\alpha = \cos^{-1}(-1/2)$

$\alpha = 120^\circ$ (Proved)

Q.9 Let

$a = 375 \quad b = 214 \quad c = 413$

$\alpha = ? \quad \beta = ? \quad \gamma = ?$



Now $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$\cos\alpha = \dfrac{(214)^2+(413)^2-(375)^2}{2(214)(413)} = 0.4285$

$\alpha = \cos^{-1}(0.4285) \Rightarrow \boxed{\alpha = 64^\circ 38'}$

Now $\cos\beta = \dfrac{c^2+a^2-b^2}{2ca}$

$\cos\beta = \dfrac{(413)^2+(375)^2-(214)^2}{2(413)(375)}$

$\cos\beta = 0.8568 \Rightarrow \beta = \cos^{-1}(0.8568)$

$\boxed{\beta = 31^\circ 2'}$

$\because \alpha+\beta+\gamma = 180^\circ$

$\gamma = 180^\circ - \alpha - \beta$

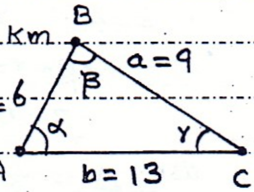
$\gamma = 180^\circ - 64^\circ 38' - 31^\circ 2'$

$\boxed{\gamma = 64^\circ 38'}$

Q.10

$a = 9\,km$ $b = 13\,km$ $c = 6\,km$

$\alpha = ?$ $\beta = ?$ $\gamma = ?$



Using

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos\alpha = \frac{(13)^2+(6)^2-(9)^2}{2(13)(6)} = 0.7949$$

$$\alpha = \cos^{-1}(0.7949)$$

$$\boxed{\alpha = 37^\circ\ 21'}$$

Now $\cos\beta = \dfrac{c^2+a^2-b^2}{2ca}$

$$\cos\beta = \frac{(6)^2+(9)^2-(13)^2}{2(6)(9)} = -0.4815$$

$$\beta = \cos^{-1}(-0.4815)$$

$$\boxed{\beta = 118^\circ\ 47'}$$

$\because\ \alpha+\beta+\gamma = 180^\circ$

$$\gamma = 180^\circ - \alpha - \beta$$

$$\gamma = 180^\circ - 37^\circ 21' - 118^\circ 47'$$

$$\boxed{\gamma = 23^\circ\ 52'}$$

AREA OF TRIANGLE:-

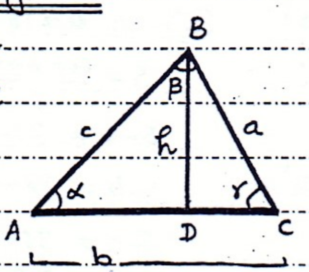
Case (1):-

When two sides and their included (3rd) angle is given:-

Consider ΔABC

Such that $\overline{BD} \perp \overline{AC}$

where $\overline{BD} = h$



We know that

Area of triangle $= \frac{1}{2}$ (Base)(Altitude)

Now from ΔABD $\sin\alpha = \dfrac{h}{c}$

$$\Rightarrow h = c\sin\alpha$$

So

$$\Delta = \frac{1}{2}(b)(h)$$

$$\Delta = \frac{1}{2}(b)(c\sin\alpha)$$

$$\boxed{\Delta = \frac{1}{2}bc\sin\alpha}$$

Similarly, we can prove

$$\boxed{\Delta = \frac{1}{2}ab\sin\gamma}$$

$$\boxed{\Delta = \frac{1}{2}ca\sin\beta}$$

Case (2):-

When one side and two angles are given:-

We know that

$$\Delta = \frac{1}{2}ab\sin\gamma \quad \text{—} \ (1)$$

Using $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow b = \frac{a}{\sin\alpha}\times\sin\beta$$

putting in (1)

$$\Delta = \frac{1}{2}a\left(\frac{a}{\sin\alpha}\times\sin\beta\right)\sin\gamma$$

$$\boxed{\Delta = \frac{1}{2}a^2\,\frac{\sin\beta\sin\gamma}{\sin\alpha}}$$

Similarly

$$\boxed{\Delta = \frac{1}{2}b^2\,\frac{\sin\gamma\sin\alpha}{\sin\beta}}$$

$$\boxed{\Delta = \frac{1}{2}c^2\,\frac{\sin\alpha\sin\beta}{\sin\gamma}}$$