Again $\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}$

$c=\frac{a}{\sin\alpha}\times\sin\gamma=\frac{89.35}{\sin 38^\circ 25'}\times\sin 89^\circ 35'$

$\boxed{c = 143.79}$

Q.3 $a=?$ $b=125$ $c=?$

$\alpha=47^\circ$ $\beta=?$ $\gamma=53^\circ$

$\because \alpha+\beta+\gamma=180^\circ$

$\beta=180^\circ-\alpha-\gamma^\circ$

$\beta=180^\circ-47^\circ-53^\circ$

$\boxed{\beta=80^\circ}$

Using $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}$

$\Rightarrow a=\frac{b}{\sin\beta}\times\sin\alpha=\frac{125}{\sin 80^\circ}\times\sin 47^\circ$

$\boxed{a= 92.83}$

Now $\frac{c}{\sin\gamma}=\frac{b}{\sin\beta}$

$c=\frac{b}{\sin\beta}\times\sin\gamma=\frac{125}{\sin 80^\circ}\times\sin 53^\circ$

$\boxed{c= 101.37}$

Q.4 $a=?$ $b=?$ $c=16.1$

$\alpha=42^\circ 45'$ $\beta=?$ $\gamma=74^\circ 32'$

$\because \alpha+\beta+\gamma=180^\circ$

$\beta=180^\circ-\alpha-\gamma$

$\beta=180^\circ-42^\circ 45'-74^\circ 32'$

$\boxed{\beta=62^\circ 43'}$

Using $\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}$

$a=\frac{c}{\sin\gamma}\times\sin\alpha=\frac{16.1}{\sin 74^\circ 32'}\times\sin 42^\circ 45'$

$\boxed{a= 11.34}$

Again $\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$

$b=\frac{c}{\sin\gamma}\times\sin\beta=\frac{16.1}{\sin 74^\circ 32'}\times\sin 62^\circ 43'$

$\boxed{b= 14.85}$

Q.5 $a=53$ $b=?$ $c=?$

$\alpha=?$ $\beta=88^\circ 36'$ $\gamma=31^\circ 54'$

$\because \alpha+\beta+\gamma=180^\circ$

$\alpha=180^\circ-\beta-\gamma$

$\alpha=180^\circ-88^\circ 36'-31^\circ 54'$

$\boxed{\alpha= 59^\circ 30'}$

Using $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}$

$b=\frac{a}{\sin\alpha}\times\sin\beta=\frac{53}{\sin 59^\circ 30'}\times\sin 88^\circ 36'$

$\boxed{b= 61.51}$

Again $\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}$

$c=\frac{a}{\sin\alpha}\times\sin\gamma=\frac{53}{\sin 59^\circ 30'}\times\sin 31^\circ 54'$

$\boxed{c= 32.51}$

"EXERCISE: 12.5"

Q.1 $a=?$ $b=95$ $c=34$

$\alpha=52^\circ$ $\beta=?$ $\gamma=?$

Using $a^2=b^2+c^2-2bc\cos\alpha$

$a^2=(95)^2+(34)^2-2(95)(34)\cos 52^\circ$

$a^2= 6203.83$

$\boxed{a= 78.76}$

Now $\cos\beta=\frac{c^2+a^2-b^2}{2ca}$

$\cos\beta=\frac{(34)^2+(78.76)^2-(95)^2}{2(34)(78.76)}$

$\cos\beta=-0.3110$

$\beta=\cos^{-1}(-0.3110)$

$\boxed{\beta= 108^\circ 7' 20''}$

$\because \alpha+\beta+\gamma=180^\circ$

$\gamma=180-\alpha-\beta$

$\gamma=180-52^\circ-108^\circ 7' 20''$

$\boxed{\gamma= 19^\circ 52' 40''}$

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Q.2 $a = ?$ $b = 12.5$ $c = 23$

$\alpha = 38^\circ 26'$ $\beta = ?$ $\gamma = ?$

Using $a^2 = b^2 + c^2 - 2bc\cos\alpha$

$a^2 = (12.5)^2 + (23)^2 - 2(12.5)(23)\cos 38^\circ 26'$

$a^2 = 234.22$

$\boxed{a = 15.30}$

Now $\cos\beta = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos\beta = \dfrac{(15.30)^2 + (23)^2 - (12.5)^2}{2(23)(15.30)}$

$\cos\beta = 0.8622$

$\beta = \cos^{-1}(0.8622)$

$\boxed{\beta = 30^\circ 26'}$

$\because \alpha + \beta + \gamma = 180^\circ$

$\gamma = 180^\circ - \alpha - \beta$

$\gamma = 180^\circ - 30^\circ 26' - 38^\circ 26'$

$\boxed{\gamma = 111^\circ 14'}$

Q.3 $a = \sqrt{3} - 1$ $b = \sqrt{3} + 1$ $c = ?$

$\alpha = ?$ $\beta = ?$ $\gamma = 60^\circ$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = (\sqrt{3}-1)^2 + (\sqrt{3}+1)^2 - 2(\sqrt{3}-1)(\sqrt{3}+1)\cos 60^\circ$

$= 3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3} - 2(3-1)\ \tfrac{1}{2}$

$= 8 - 2 = 6$

$c^2 = 6$

$\boxed{c = \sqrt{6}}$

Using $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$

$\sin\alpha = \dfrac{a}{c} \times \sin\gamma$

$\sin\alpha = \dfrac{\sqrt{3}-1}{\sqrt{6}} \times \sin 60^\circ$

$\sin\alpha = 0.2588$

$\alpha = \sin^{-1}(0.2588)$

$\boxed{\alpha = 15^\circ}$

$\because \alpha + \beta + \gamma = 180^\circ$

$\beta = 180^\circ - \alpha - \gamma$

$\beta = 180 - 15^\circ - 60^\circ$

$\boxed{\beta = 105^\circ}$

Q.4 $a = 3$ $b =$ $c = 6$

$\alpha =$ $\beta = 36^\circ 20'$ $\gamma =$

Using $b^2 = c^2 + a^2 - 2ca\cos\beta$

$b^2 = (6)^2 + (3)^2 - 2(6)(3)\cos 36^\circ 20'$

$b^2 = 16 \Rightarrow \boxed{b = 4}$

Using $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos\alpha = \dfrac{(4)^2 + (6)^2 - (3)^2}{2(4)(6)} = 0.8958$

$\alpha = \cos^{-1}(0.8958)$

$\boxed{\alpha = 26^\circ 23'}$

$\because \alpha + \beta + \gamma = 180^\circ$

$\gamma = 180^\circ - \alpha - \beta$

$\gamma = 180^\circ - 26^\circ 23' - 36^\circ 20'$

$\boxed{\gamma = 117^\circ 17'}$

Q.5 $a = 7$ $b = 3$ $c = ?$

$\alpha = ?$ $\beta = ?$ $\gamma = 38^\circ 13'$

Using $c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = (7)^2 + (3)^2 - 2(7)(3)\cos 38^\circ 13'$

$c^2 = 25.002$

$\boxed{c = 5}$

Now $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$

$\Rightarrow \sin\alpha = \dfrac{a}{c} \times \sin\gamma = \dfrac{7}{5}\sin 38^\circ 13'$

$\sin\alpha = 0.8661$

$\alpha = \sin^{-1}(0.8661)$

$\boxed{\alpha = 60^\circ}$

$\because \alpha+\beta+\gamma = 180^\circ$

So $\beta = 180^\circ - \alpha - \gamma$

$\beta = 180^\circ - 60^\circ - 38^\circ 13'$

$\boxed{\beta = 81^\circ 47'}$

Solve Ist using Law of tangents then Law of Sines.

Q.6 $a = 36.21$ $b = 42.09$ $c = ?$

$\alpha = ?$ $\beta = ?$ $\gamma = 44^\circ 29'$

$\because \alpha+\beta+\gamma = 180^\circ$

$\alpha+\beta = 180^\circ - \gamma = 180^\circ - 44^\circ 29'$

$\alpha+\beta = 135^\circ 31'$ —— ①

Now $\dfrac{a-b}{a+b} = \dfrac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$

$\dfrac{36.21-42.09}{36.21+42.09} = \dfrac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{135^\circ 31'}{2}\right)}$

$\tan\left(\dfrac{\alpha-\beta}{2}\right) = \dfrac{-5.88}{78.30} \times \tan\left(\dfrac{135^\circ 31'}{2}\right)$

$\tan\left(\dfrac{\alpha-\beta}{2}\right) = -0.1836$

$\dfrac{\alpha-\beta}{2} = -10^\circ 24' 20''$

$\alpha-\beta = -20^\circ 48' 40''$ —— ②

Adding ① and ② $2\alpha = 114^\circ 42'$

$\boxed{\alpha = 57^\circ 21'}$

Now ① $\Rightarrow$ $\beta = 135^\circ 31' - \alpha$

$\beta = 135^\circ 31' - 57^\circ 21'$

$\boxed{\beta = 78^\circ 10'}$

Now $\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

$c = \dfrac{b}{\sin\beta} \times \sin\gamma$

$c = \dfrac{42.09}{\sin 78^\circ 10'} \times \sin 44^\circ 29'$

$\boxed{c = 30.13}$

Q.7 $a = 93$ $b = ?$ $c = 101$

$\alpha = ?$ $\beta = 80^\circ$ $\gamma = ?$

$\because \alpha+\beta+\gamma = 180^\circ$

$\alpha+\gamma = 180^\circ - \beta = 180^\circ - 80^\circ$

$\gamma+\alpha = 100^\circ$ —— ①

Now $\dfrac{c-a}{c+a} = \dfrac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{\gamma+\alpha}{2}\right)}$

$\dfrac{101-93}{101+93} = \dfrac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{100^\circ}{2}\right)}$

$\tan\left(\dfrac{\gamma-\alpha}{2}\right) = \dfrac{8}{194} \times \tan 50^\circ$

$\tan\left(\dfrac{\gamma-\alpha}{2}\right) = 0.0491$

$\dfrac{\gamma-\alpha}{2} = 2^\circ 49'$

$\gamma-\alpha = 5^\circ 38'$ —— ②

Adding ① and ②

$2\gamma = 105^\circ 38'$

$\boxed{\gamma = 52^\circ 49'}$

① $\Rightarrow$ $\alpha = 100^\circ - \gamma$

$\alpha = 100^\circ - 52^\circ 49'$

$\boxed{\alpha = 47^\circ 11'}$

Now $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$b = \dfrac{a}{\sin\alpha} \times \sin\beta$

$b = \dfrac{93}{\sin 47^\circ 11'} \times \sin 80^\circ$

$\boxed{b = 124.85}$

Q.8 $a = ?$ $b = 14.8$ $c = 16.1$

$\alpha = 42^\circ 45'$ $\beta = ?$ $\gamma = ?$

$\because \alpha+\beta+\gamma = 180^\circ$

$\beta+\gamma = 180^\circ - \alpha = 180^\circ - 42^\circ 45'$

$\beta+\gamma = 137^\circ 15'$ —— ①

Using $\dfrac{b-c}{b+c} = \dfrac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)}$

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$$\frac{14.8-16.1}{14.8+16.1} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{137^\circ 15'}{2}\right)}$$

$$\frac{-1.3}{30.9} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{137^\circ 15'}{2}\right)}$$

$$\tan\left(\frac{\beta-\gamma}{2}\right) = \frac{-1.3}{30.9} \times \tan\left(\frac{137^\circ 15'}{2}\right)$$

$$\tan\left(\frac{\beta-\gamma}{2}\right) = -0.1075$$

$$\frac{\beta-\gamma}{2} = -6^\circ 8'$$

$$\beta-\gamma = -12^\circ 16' \quad \text{—— (2)}$$

Adding (1) & (2) $\Rightarrow 2\beta = 124^\circ 59^\circ$

$$\boxed{\beta = 62^\circ 29'}$$

(1) $\Rightarrow \gamma = 137^\circ 15' - 62^\circ 29'$

$$\boxed{\gamma = 74^\circ 46'}$$

Now $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$$a = \frac{b}{\sin\beta} \times \sin\alpha$$

$$a = \frac{14.8}{\sin 62^\circ 29'} \times \sin 42^\circ 45'$$

$$\boxed{a = 11.33}$$

Q.9 $a = 319 \quad b = 168 \quad c = ?$

$\alpha = ? \quad \beta = ? \quad \gamma = 110^\circ 22'$

$\because \alpha+\beta+\gamma = 180^\circ$

$$\alpha+\beta = 180^\circ - \gamma = 180^\circ - 110^\circ 22'$$

$$\alpha+\beta = 69^\circ 38' \quad \text{—— (1)}$$

Now $\dfrac{a-b}{a+b} = \dfrac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$

$$\frac{319-168}{319+168} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{69^\circ 38'}{2}\right)}$$

$$\tan\left(\frac{\alpha-\beta}{2}\right) = \frac{151}{487} \times \tan\left(\frac{69^\circ 38'}{2}\right)$$

$$\tan\left(\frac{\alpha-\beta}{2}\right) = 0.2156$$

$$\frac{\alpha-\beta}{2} = 12^\circ 10'$$

$$\alpha-\beta = 24^\circ 20' \quad \text{—— (2)}$$

Adding (1) and (2) $2\alpha = 93^\circ 58'$

$$\boxed{\alpha = 46^\circ 59'}$$

(1) $\Rightarrow \beta = 69^\circ 38' - 46^\circ 59'$

$$\boxed{\beta = 22^\circ 39'}$$

Using Law of Sines:

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$c = \frac{a}{\sin\alpha} \times \sin\gamma$$

$$c = \frac{319}{\sin 46^\circ 59'} \times \sin 110^\circ 22'$$

$$\boxed{c = 409.02}$$

Q.10 $a = 32 \quad b = 61 \quad c = ?$

$\alpha = ? \quad \beta = ? \quad \gamma = 59^\circ 30'$

$\because \alpha+\beta+\gamma = 180^\circ$

$$\alpha+\beta = 180^\circ - \gamma = 180^\circ - 59^\circ 30'$$

$$\alpha+\beta = 120^\circ 30' \quad \text{—— (1)}$$

Using $\dfrac{a-b}{a+b} = \dfrac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$

$$\frac{32-61}{32+61} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{120^\circ 30'}{2}\right)}$$

$$\tan\left(\frac{\alpha-\beta}{2}\right) = \frac{-31}{93} \times \tan\left(\frac{120^\circ 30'}{2}\right)$$

$$\tan\left(\frac{\alpha-\beta}{2}\right) = -0.5832$$

$$\frac{\alpha-\beta}{2} = -30^\circ 15'$$

$$\alpha-\beta = -60^\circ 30' \quad \text{—— (2)}$$

Adding (1) & (2) $2\alpha = 60^\circ$

$$\boxed{\alpha = 30^\circ}$$

(1) $\Rightarrow \beta = 120^\circ 30' - 30^\circ$

$$\boxed{\beta = 90^\circ 30'}$$

Using Law of Sines:

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$c = \frac{a}{\sin\alpha} \times \sin\gamma$$

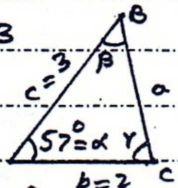
$$c = \frac{32}{\sin 30^\circ} \times \sin 59^\circ 30'$$

$$\boxed{c = 55.14}$$

Q.11 Let the Sides are 2 and 3

$a = ?$ $b = 2$ $c = 3$

$\alpha = 57^\circ$ $\beta = ?$ $\gamma = ?$



Using $\alpha + \beta + \gamma = 180^\circ$

$$\beta + \gamma = 180^\circ - 57^\circ$$

$$\beta + \gamma = 123^\circ \quad \text{---} \quad (1)$$

Now $$\frac{b-c}{b+c} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)}$$

$$\frac{2-3}{2+3} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{123^\circ}{2}\right)}$$

$$\tan\left(\frac{\beta-\gamma}{2}\right) = \frac{-1}{5} \times \tan\left(\frac{123^\circ}{2}\right)$$

$$\tan\left(\frac{\beta-\gamma}{2}\right) = -0.3684$$

$$\frac{\beta-\gamma}{2} = -20^\circ 13'$$

$$\beta - \gamma = -40^\circ 26' \quad \text{---} \quad (2)$$

Adding (1) and (2)

$$2\beta = 82^\circ 34'$$

$$\boxed{\beta = 41^\circ 17'}$$

$(1) \Rightarrow \gamma = 123^\circ - \beta$

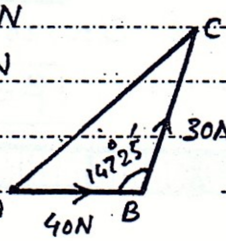
$$\gamma = 123^\circ - 41^\circ 17'$$

$$\boxed{\gamma = 81^\circ 43'}$$

Q.12 $|\vec{AB}| = 40N$

$|\vec{BC}| = 30N$

Angle b/w forces $= 147^\circ 25'$



Using Law of Cosines:

$$(\vec{AC})^2 = (\vec{AB})^2 + (\vec{BC})^2 - 2(\vec{AB})(\vec{BC})\cos 147^\circ 25'$$

$$(\vec{AC})^2 = (40)^2 + (30)^2 - 2(40)(30)\cos(147^\circ 25')$$

$$(\vec{AC})^2 = 4522.26$$

$$|\vec{AC}| = 67.25\,N$$

EXERCISE: 12.6

Q.1 $a = 7$ $b = 7$ $c = 9$

$\alpha = ?$ $\beta = ?$ $\gamma = ?$

Using $$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{7^2 + 9^2 - 7^2}{2(7)(9)} = 0.6429$$

$$\alpha = \cos^{-1}(0.6429)$$

$$\boxed{\alpha = 50^\circ}$$

Using $$\cos\beta = \frac{c^2 + a^2 - b^2}{2ca}$$

$$= \frac{9^2 + 7^2 - 7^2}{2(9)(7)} = 0.6429$$

$$\beta = \cos^{-1}(0.6429)$$

$$\boxed{\beta = 50^\circ}$$

Now $\alpha + \beta + \gamma = 180^\circ$

$$\gamma = 180^\circ - 50^\circ - 50^\circ$$

$$\boxed{\gamma = 80^\circ}$$

Q.2 $a = 32$ $b = 40$ $c = 66$

$\alpha = ?$ $\beta = ?$ $\gamma = ?$

Using $$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(40)^2 + (66)^2 - (32)^2}{2(40)(66)}$$

$$\cos\alpha = 0.9341$$

$$\alpha = \cos^{-1}(0.9341)$$

$$\boxed{\alpha = 20^\circ 55'}$$