F.Sc.(Part-1)
TRIGONOMETRY
CH # 9---14

IMPORTANT—FORMULAE

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Relation between ℓ and θ

$$\ell = r\theta$$

where θ is in Radian, ℓ and r measured in terms of same unit.

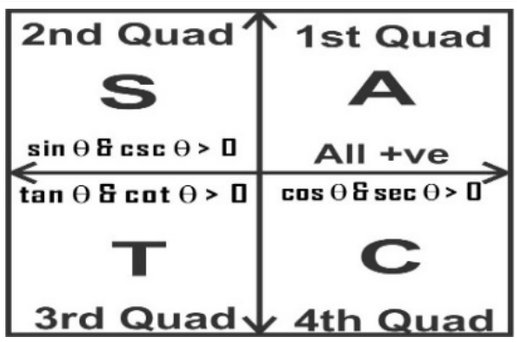
Conversion of Degree $\Leftrightarrow$ Radian

$$1^\circ = \frac{\pi}{180}\text{rad} \quad \& \quad 1\text{rad} = \frac{180^\circ}{\pi}$$

Fundamental Identities

1. $\sin^2\theta + \cos^2\theta = 1$ or $\begin{cases}\sin^2\theta = 1-\cos^2\theta \\ \cos^2\theta = 1-\sin^2\theta\end{cases}$
2. $1+\tan^2\theta = \sec^2\theta$ or $\begin{cases}\tan^2\theta = \sec^2\theta - 1 \\ \sec^2\theta - \tan^2\theta = 1\end{cases}$
3. $1+\cot^2\theta = \operatorname{cosec}^2\theta$ or $\begin{cases}\cot^2\theta = \operatorname{cosec}^2\theta - 1 \\ \operatorname{cosec}^2\theta - \cot^2\theta = 1\end{cases}$
4. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ &
5. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$
6. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ &
7. $\sec\theta = \dfrac{1}{\cos\theta}$

Signs of Trigonometric Functions



Values of Trigonometric Functions

θ	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Fundamental Laws of Trigonometry

1. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
2. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
3. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
4. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
5. $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
6. $\tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Double Angles Identities

1. $\sin 2\theta = 2\sin\theta\cos\theta$
2. $\cos 2\theta = \begin{cases}\cos^2\theta - \sin^2\theta \\ 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta\end{cases}$
3. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Triple Angles Identities

1. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
2. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
3. $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

Sum & Difference $\Leftrightarrow$ Product

1. $2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$
2. $2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$
3. $2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$
4. $-2\sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$
5. $\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
6. $\sin A - \sin B = 2\cos\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$
7. $\cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
8. $\cos A - \cos B = -2\sin\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$

Domain & Range of Trigonometric Functions

Functions	Domain	Range
$y = \sin x$	$-\infty < x < +\infty$	$-1 \le y \le 1$
$y = \cos x$	$-\infty < x < +\infty$	$-1 \le y \le 1$
$y = \tan x$	$-\infty < x < +\infty$, $x \neq \frac{(2n+1)\pi}{2}$	$-\infty < y < +\infty$
$y = \cot x$	$-\infty < x < +\infty$, $x \neq \frac{(2n+1)\pi}{2}$	$-\infty < y < +\infty$
$y = \sec x$	$-\infty < x < +\infty$, $x \neq n\pi$	$y \ge 1$ *or* $y \le -1$
$y = \csc x$	$-\infty < x < +\infty$, $x \neq n\pi$	$y \ge 1$ *or* $y \le -1$

Period of Trigonometric Functions

Function	Period
$\sin a\theta$, $\cos a\theta$, $\csc a\theta$, $\sec a\theta$	$\frac{2\pi}{a}$
$\tan a\theta$, $\cot a\theta$	π

The Law of Sines

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$$

The Law of Cosines

1. $a^2=b^2+c^2-2bc\cos\alpha$
2. $b^2=c^2+a^2-2ca\cos\beta$
3. $c^2=a^2+b^2-2ab\cos\gamma$

The Law of Tangents

1. $\frac{a-b}{a+b}=\frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$
2. $\frac{b-c}{b+c}=\frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)}$
3. $\frac{c-a}{c+a}=\frac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{\gamma+\alpha}{2}\right)}$

Half Angle Formulas

1. $\sin\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$
2. $\sin\frac{\beta}{2}=\sqrt{\frac{(s-c)(s-a)}{ca}}$
3. $\sin\frac{\gamma}{2}=\sqrt{\frac{(s-a)(s-b)}{ab}}$
4. $\cos\frac{\alpha}{2}=\sqrt{\frac{s(s-a)}{bc}}$
5. $\cos\frac{\beta}{2}=\sqrt{\frac{s(s-b)}{ca}}$
6. $\cos\frac{\gamma}{2}=\sqrt{\frac{s(s-c)}{ab}}$
7. $\tan\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
8. $\tan\frac{\beta}{2}=\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$
9. $\tan\frac{\gamma}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

Area Of Triangles

1. $\Delta=\frac{1}{2}bc\sin\alpha=\frac{1}{2}ca\sin\beta=\frac{1}{2}ab\sin\gamma$
2. $\Delta=\frac{a^2\sin\beta\sin\gamma}{2\sin\alpha}=\frac{b^2\sin\gamma\sin\alpha}{2\sin\beta}=\frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$
3. $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$

Circum-Circle (R)

1. $R=\frac{a}{2\sin\alpha}=\frac{b}{2\sin\beta}=\frac{c}{2\sin\gamma}$
2. $R=\frac{abc}{2\Delta}$

In-Circle (r)

$$r=\frac{\Delta}{s}$$

Escribed-Circle (r_1, r_2, r_3)

$$r_1=\frac{\Delta}{s-a},\ r_2=\frac{\Delta}{s-b},\ r_3=\frac{\Delta}{s-c}$$

Inverse Trigonometric Formulas

1. $\sin^{-1}A+\sin^{-1}B=\sin^{-1}\left(A\sqrt{1-B^2}+B\sqrt{1-A^2}\right)$
2. $\sin^{-1}A-\sin^{-1}B=\sin^{-1}\left(A\sqrt{1-B^2}-B\sqrt{1-A^2}\right)$
3. $\cos^{-1}A+\cos^{-1}B=\cos^{-1}\left(AB-\sqrt{(1-A^2)(1-B^2)}\right)$
4. $\cos^{-1}A-\cos^{-1}B=\cos^{-1}\left(AB+\sqrt{(1-A^2)(1-B^2)}\right)$
5. $\tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$
6. $\tan^{-1}A-\tan^{-1}B=\tan^{-1}\left(\frac{A-B}{1+AB}\right)$
7. $2\tan^{-1}A=\tan^{-1}\left(\frac{2A}{1-A^2}\right)$

Domain & Range of Principal Trigonometric Functions

Function	Domain	Range
$y=sinx$	$-\frac{\pi}{2}\le x\le\frac{\pi}{2}$	$-1\le y\le 1$
$y=\sin^{-1}x$	$-1\le y\le 1$	$-\frac{\pi}{2}\le x\le\frac{\pi}{2}$
$y=cosx$	$0\le x\le\pi$	$-1\le y\le 1$
$y=\cos^{-1}x$	$-1\le y\le 1$	$0\le x\le\pi$
$y=tanx$	$-\frac{\pi}{2}<x<\frac{\pi}{2}$	R
$y=\tan^{-1}x$	R	$-\frac{\pi}{2}<x<\frac{\pi}{2}$
$y=cotx$	$0<x<\pi$	R
$y=\cot^{-1}x$	R	$0<x<\pi$
$y=secx$	$[0,\pi], x\neq\frac{\pi}{2}$	$y\le -1\ or\ y\ge 1$
$y=\sec^{-1}x$	$x\le -1\ or\ x\ge 1$	$[0,\pi], y\neq\frac{\pi}{2}$
$y=\csc x$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right], x\neq 0$	$y\le -1\ or\ y\ge 1$
$y=\csc^{-1}x$	$x\le -1\ or\ x\ge 1$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right], y\neq 0$

Best of Luck

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