DAE (1st Year)
Math-113
CH # 01---19

IMPORTANT—FORMULAE

By: Ali Nawaz Bajwa (M.Phil. (Math), M.Ed.) Mob # : +92(345)6743869
Lecturer in Mathematics Government College of Technology, Sialkot.

Chapter # 01

Exercise # 1.1

Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Exercise # 1.2

Discriminant

$$\text{Disc}=b^2-4ac$$

Exercise # 1.3

Sum & Product of the Roots

$$\text{Sum of the Roots}=\alpha+\beta=-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}=-\frac{b}{a}$$

$$\text{Product of the Roots}=\alpha\beta=\frac{\text{constant term}}{\text{coefficient of } x^2}=\frac{c}{a}$$

Exercise # 1.4

Formation of Quadratic Equation

$$x^2-Sx+P=0$$

Where $\left(S = \text{Sum of the roots} \; \& \; P = \text{Product of the roots.}\right)$

Chapter # 02

Exercise # 2.1

n^{th} term or General term of A.P.

$$a_n=a+(n-1)d$$

Where $\left(a = 1^{st}\text{ term}, d = \text{common difference} \; \& \; n = \text{No. of terms.}\right)$

Exercise # 2.2

Arithmetic Mean (AM)

$$A=\frac{a+b}{2}$$

Exercise # 2.3

Sum of 'n' terms of an Arithmetic Series

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

Exercise # 2.4

n^{th} term or General term of G.P.

$$a_n=ar^{n-1}$$

Where $\left(a = 1^{st}\text{ term}, d = \text{common Ratio} \; \& \; n = \text{No. of terms.}\right)$

Exercise # 2.5

Geometric Mean (GM)

$$G=\pm\sqrt{ab}$$

Exercise # 2.6

Sum of 'n' terms of a Geometric Series

$$S_n=\begin{cases}\dfrac{a(1-r^n)}{1-r} & \text{if} \quad |r|<1\\[2mm] \dfrac{a(r^n-1)}{r-1} & \text{if} \quad |r|>1\end{cases}$$

Exercise # 2.7

Sum of '∞' terms of a Geometric Series

$$S_\infty=\begin{cases}\dfrac{a}{1-r} & \text{if} \quad |r|<1\\[2mm] \dfrac{a}{r-1} & \text{if} \quad |r|>1\end{cases}$$

Chapter # 03

Exercise # 3.1

Binomial Theorem

1. $n!=n(n-1)(n-2)\ldots 3.2.1$
2. ${}^{n}C_{r}=\binom{n}{r}=\dfrac{n!}{r!(n-r)!}$
3. Binomial Theorem

$$(a+b)^n=\binom{n}{0}a^nb^o+\binom{n}{1}a^{n-1}b^1+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{n}a^ob^n$$

Exercise # 3.2

Binomial Series

$$(1+b)^n=1+\frac{n}{1!}b+\frac{n(n-1)}{2!}b^2+\frac{n(n-1)(n-2)}{3!}b^3+\ldots$$

Chapter # 04

How to resolve a Rational Fraction into Partial Fraction

1. The degree of N(x) must be less than that of degree of D(x). if not, divide and work with the remainder theorem.
2. Multiply both sides by D(x).
3. Equate the coefficients of like powers of x.
4. solve the resulting equations for the coefficients.

Chapter # 05

Exercise # 5.1

Relation between ℓ and θ

$$\ell=r\theta$$

where θ is in Radian, ℓ and r measured in terms of same unit.

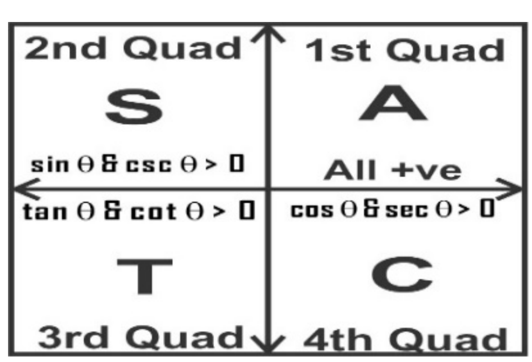
Exercise # 5.2

Conversion of Degree ⇔ Radian

$$1^\circ=\frac{\pi}{180}\text{rad} \quad \& \quad 1\text{rad}=\frac{180^\circ}{\pi}$$

Exercise # 5.3

Signs of Trigonometric Functions



Values of Trigonometric Functions

θ	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Exercise # 5.3

Fundamental Identities

1. $\sin^2\theta+\cos^2\theta=1$ or $\begin{cases}\sin^2\theta=1-\cos^2\theta\\ \cos^2\theta=1-\sin^2\theta\end{cases}$

2. $1+\tan^2\theta=\sec^2\theta$ or $\begin{cases}\tan^2\theta=\sec^2\theta-1\\ \sec^2\theta-\tan^2\theta=1\end{cases}$

3. $1+\cot^2\theta=\operatorname{cosec}^2\theta$ or $\begin{cases}\cot^2\theta=\operatorname{cosec}^2\theta-1\\ \operatorname{cosec}^2\theta-\cot^2\theta=1\end{cases}$

4. $\tan\theta=\frac{\sin\theta}{\cos\theta}$ & **5.** $\cot\theta=\frac{\cos\theta}{\sin\theta}$

6. $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ & **7.** $\sec\theta=\frac{1}{\cos\theta}$

Chapter # 06

Exercise # 6.1

Fundamental Laws of Trigonometry

1. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

2. $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

3. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

4. $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

5. $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

6. $\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Exercise # 6.2

Double Angles Identities

1. $\sin 2\theta=2\sin\theta\cos\theta$

2. $\cos 2\theta=\begin{cases}\cos^2\theta-\sin^2\theta\\ 2\cos^2\theta-1\\ 1-2\sin^2\theta\end{cases}$

3. $\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$

Triple Angles Identities

1. $\sin 3\theta=3\sin\theta-4\sin^3\theta$

2. $\cos 3\theta=4\cos^3\theta-3\cos\theta$

3. $\tan 3\theta=\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$

Exercise # 6.3

Product ⇒ Sum & Difference

1. $2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$

2. $2\cos\alpha\sin\beta=\sin(\alpha+\beta)-\sin(\alpha-\beta)$

3. $2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$

4. $-2\sin\alpha\sin\beta=\cos(\alpha+\beta)-\cos(\alpha-\beta)$

Sum & Difference ⇒ Product

1. $\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

2. $\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

3. $\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

4. $\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

Chapter # 07

Exercise # 7.1 , 7.2

Right Angle Triangle

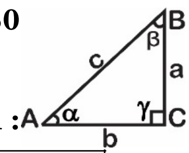
1. $\sin\alpha=\frac{\text{Perpendicular}}{\text{Hypotenuse}}=\frac{a}{c}=$ عمود / وتر (عو)

2. $\cos\alpha=\frac{\text{Base}}{\text{Hypotenuse}}=\frac{b}{c}=$ قاعدہ / وتر (قو)

3. $\tan\alpha=\frac{\text{Perpendicular}}{\text{Base}}=\frac{a}{b}=$ عمود / قاعدہ (عق)

4. Sum of angles of the triangle is 180

$\alpha+\beta+\gamma=180°$



5. The Pythagoras Theorem:

$(\text{Base})^2+(\text{Perpendicular})^2=(\text{Hypotenus})^2$

Exercise # 7.3, 7.4, 7.5

Oblique Triangles

1. Sum of angles of a triangle: $\alpha+\beta+\gamma=180°$

2. The Law of Sines: $\left\{\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}\right\}$

3. The Law of cosines:
- **i.** $a^2=b^2+c^2-2bc\cos\alpha$
- **ii.** $b^2=c^2+a^2-2ca\cos\beta$
- **iii.** $c^2=a^2+b^2-2ab\cos\gamma$

Chapter # 08

Exercise # 8.1

1. Magnitude of a vector $\vec{r}=xi+yj+zk$ is

$$|\vec{r}|=\sqrt{x^2+y^2+z^2}$$

2. Units vector of a vector $\vec{a}=\hat{a}=\frac{\vec{a}}{|\vec{a}|}$

3. Direction Cosines of a vector $\vec{r}=xi+yj+zk$ are

$$\frac{x}{\sqrt{x^2+y^2+z^2}},\ \frac{y}{\sqrt{x^2+y^2+z^2}},\ \frac{z}{\sqrt{x^2+y^2+z^2}}$$

Exercise # 8.2

1. Cosine of the angle between $\vec{a}$ & $\vec{b}=\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$

2. Sine of the angle between $\vec{a}$ & $\vec{b}=\sin\theta=\frac{|\vec{a}\times\vec{b}|}{|\vec{a}|\ |\vec{b}|}$

3. Units vector perpendicular to both $\vec{a}$ & $\vec{b}=\hat{u}=\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

4. Area of the Parallelogram $=|\vec{a}\times\vec{b}|$

5. Two vectors $\vec{a}$ & $\vec{b}$ are perpendicular iff $\vec{a}.\vec{b}=0$

6. Two vectors $\vec{a}$ & $\vec{b}$ are parallel iff $\vec{a}\times\vec{b}=0$

7. Projection of $\vec{a}$ along $\vec{b}=\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

Chapter # 09

Exercise # 9.1

Matrices

- Two matrices A and B can be added/Subtracted if A and B have same order.
- Two matrices A and B are said to be conformable for the product AB if 'The number of columns of A = The number of rows of B'.
- Two matrices A and B are commute if and only if $AB = BA$

Exercise # 9.2

Determinant

1. If a square matrix has two rows (or columns) are identical, then $|A| = 0$
2. If all the entries of a row (or column) of a square matrix are zero then, $|A| = 0$
3. For a square matrix A, $|A| = |A^t|$.

Exercise # 9.3

Special MAtrices

- A matrix A is singular if $|A| = 0$.
- A matrix A is non-singular if $|A| \neq 0$
- A square matrix A is called symmetric if $A^t = A$
- A square matrix A is called skew symmetric matrix if $A^t = -A$
- If A is a non-singular matrix, then $A^{-1} = \dfrac{adj(A)}{|A|}$

Chapter # 10

Area of Triangle

1. When all sides are equal (Equilateral Triangle): Area of triangle $= \dfrac{\sqrt{3}}{4}a^2$
2. When two sides and their included angle are given: Area of triangle $= \dfrac{1}{2}ab\sin\theta$
3. When base and height are given: Area of triangle $= \dfrac{1}{2} \times \text{base} \times \text{height}$
4. When all sides are given (Hero's Formula): Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Chapter # 11

Area of Quadrilateral

1. Area of square $= a^2$
2. Area of Rhombus $= \begin{cases} \dfrac{d_1 \times d_2}{2} \\ a^2\sin\theta \end{cases}$
3. Area of Rectangle = base × height = b × h
4. Area of Parallelogram $= \begin{cases} ab\sin\theta \\ \text{base} \times \text{Altitude} \end{cases}$
5. Area of Trapezoid $= \dfrac{\text{sum of Parallel sides}}{2} \times \text{height}$
6. Area of Quadrilateral $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$

Chapter # 12

Area of Polygons

1. Regular Polygon of 'n' sides: Interior angle $= \dfrac{2n-4}{n} \times 90°$
2. When length of a side 'a' is given: Area $= \dfrac{na^2}{4}\cot\left(\dfrac{180°}{n}\right)$
3. When radius of the inscribed cicle 'r' is given: Area $= nr^2\tan\left(\dfrac{180°}{n}\right)$
4. When radius of the circumscribed cicle 'R' is given: Area $= \dfrac{nR^2}{2}\sin\left(\dfrac{360°}{n}\right)$

Chapter # 13

Area of Circles

1. Area of Circle $= A = \pi r^2$
2. Circumference of Circle $= C = 2\pi r$
3. Area of annulus(Ring) $= \pi(R^2 - r^2)$
4. Area of Sector $= \begin{cases} \dfrac{1}{2}r^2\theta \\ \dfrac{1}{2}\ell r \end{cases}$
5. Area of $\begin{cases} \text{minor Segment} = \dfrac{1}{2}r^2(\theta - \sin\theta) \\ \text{major Segment} = \dfrac{1}{2}r^2(2\pi - \theta + \sin\theta) \end{cases}$
6. Area of Segment $= \dfrac{h}{6c}[3h^2 + 4c^2]$
7. Length of chord $= C = \sqrt{dh - h^2}$

Chapter # 14

Area of Irregular Plane Figures

Simpson's Rule

Area $= \dfrac{S}{3}[A + 2D + 4E]$

- S = Width of each strip
- A = Sum of first & last ordinates
- D = Sum of odd ordinates
- E = Sum of even ordinates

Chapter # 15

Mensuration of Solid

1. Lateral surface area (L.S.A.) = Perimeter of base × Height
2. Total surface area = L.S.A. + Areas of the bases
3. Volume of Prism = Area of base × Height
4. Length of diagonal $= \sqrt{\ell^2 + b^2 + h^2}$

Chapter # 16

Mensuration of Cylinder

Cylinder:
1. Lateral surface area (L.S.A.) $= 2\pi rh$
2. Total surface area $= 2\pi rh + 2\pi r^2$
3. Volume $= \pi r^2 h$

Hollow Cylinder:
4. Lateral surface area $= 2\pi(R + r)h$
5. T.S.A. $= 2\pi(R + r)h + 2\pi(R^2 - r^2)$
6. Volume $= \pi r^2 h$

Chapter # 17

Exercise # 17.1

Mensuration of Pyramid

1. Volume of Pyramid $= \frac{1}{3} \times$ area of base $\times$ height
2. Leteral Surface area $(\mathrm{L.S.A}) = \frac{1}{2} \times$ Perimeter of the base $\times$ slant height
3. Total Surface area $(\mathrm{T.S.A}) = \mathrm{L.S.A} +$ area of base

Exercise # 17.2

Frustum of Pyramid

1. Volume of the frustum of a Pyramid $= \frac{h}{3} \times \left[A_1 + A_2 + \sqrt{A_1A_2}\right]$

   (where A_1 is the base & A_2 is the area of top)
2. Leteral Surface area of the frustum of a Pyramid $(\mathrm{L.S.A.})$

   $= \frac{1}{2} \times$ sum of perimeter of the base and top $\times$ slant height
3. Total Surface area $= \mathrm{L.S.A} +$ areas of base and top

Chapter # 18

Exercise # 18.1

Mensuration of Cone

1. Volume of Cone $= \frac{1}{3}\pi r^2 h$ cu.unit
2. Lateral surface area of Cone $= \pi r \ell$ sq.unit
3. Total surface area of Cone $= \pi r(\ell + r)$ sq.unit

Exercise # 18.2

Frustum of Cone

1. Volume of the frustum of a Cone $= \frac{\pi h}{3} \times \left[R^2 + r^2 + Rr\right]$ cu.unit

   (where R is the radius of the base & r is the radius of the top)
2. Leteral Surface area $(\mathrm{L.S.A})$ of Cone $= \pi(R + r)\ell$ sq.unit
3. Total Surface area $(\mathrm{T.S.A})$ of Cone $= \pi(R + r)\ell + \pi R^2 + \pi r^2$ sq.unit

Chapter # 19

Exercise # 19.1

Mensuration of Sphere

1. Volume of the Sphere $= V = \frac{4}{3}\pi r^3$ or $\frac{\pi}{6}d^3$ cu.unit
2. Surface area of the sphere $= 4\pi r^2$ or πd^2 sq.unit
3. Volume of the Shell $= \frac{4}{3}\pi(R^3 - r^3)$ or $\frac{\pi}{6}(D^3 - d^3)$ cu.unit

Exercise # 19.2

Zone (Frustum) of Sphere

1. Volume of the Zone $= V = \frac{\pi h}{6}(h^2 + 3r_1^2 + 3r_2^2)$ cu.unit
2. Total surface area of the Zone $= 2\pi rh + \pi r_1^2 + \pi r_2^2$ sq.unit
3. Surface area of the sphere $= 2\pi rh$ sq.unit
4. Volume of the segment of sphere $= V = \frac{\pi h}{6}(h^2 + 3r_1^2)$ cu.unit